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RESEARCH ARTICLE

FRACTIONAL q-DERIVATIVE OF ROBOTNOV AND HARTLEY FUNCTION

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Abstract:

In present paper, we have derived the fractional q-derivative of special functions. To begin with the theorem on term by term q-fractional differentiation has been derived. The result is an extension of an earlier result due to Yadav and Purohit [8] and Sharma and Jain [9]. As a special case, of fractional q-differentiation of Robotnov and Hartley's function has been obtained.

Key Words: Fractional integral and derivative operators, Fractional q-derivative, Generalized Robotnov and Hartley's function and Special functions.

Mathematics Subject Classification— Primary33A30, Secondary 33A25, 83C99.

Definition:

1.1. q-Analogue of Differential Operator

Al-Salam [3], has given the q-analogue of differential operator as

$$D_q f(x) = \frac{f(xq) - f(x)}{x(q-1)} \tag{1.1}$$

This is an inverse of the q-integral operator defined as

$$\int_x^{\infty} f(t)\, d(t:q) = x(1-q) \sum_{k=1}^{\infty} q^{-k} f(xq^{-k}) \tag{1.2}$$

WHERE $0 < |q| < 1$

1.2. FRACTIONAL Q-DERIVATIVE OF ORDER α:

THE FRACTIONAL Q-DERIVATIVE OF ORDER α IS DEFINED AS

$$D_{x,q}^{\alpha} f(x) = \frac{1}{\Gamma_Q(-\alpha)} \int_0^x (x - yq)_{-\alpha-1} f(y) d(y;q) \tag{1.2.1}$$

WHERE RE $(\alpha) < 0$

AS A PARTICULAR CASE OF (3), WE HAVE

$$D_{x,q}^{\alpha} x^{\mu-1} = \frac{\Gamma_Q(\mu)}{\Gamma_Q(\mu-\alpha)} x^{\mu-\alpha-1} \tag{1.2.2}$$

1.3 Robotnov and Hartley Function:

The following function was introduced (Hartley and Lorenzo, 1998) during solving of the fundamental linear fractional order differential equation:

$$F_{\alpha}[a,z] = \sum_{k=0}^{\infty} \frac{(a)^k z^{(k+1)\alpha-1}}{\Gamma(k\alpha+\alpha)}, \;\; q > 0 \tag{1.3.1}$$

This function had been studied by Robotnov (1969, 1980) with respect to hereditary integrals for application to solid machanics.

2. MAIN RESULTS

IN THIS SECTION WE DRIVE THE RESULTS ON TERM BY TERM Q-FRACTIONAL DIFFERENTIATION OF A POWER SERIES. AS PARTICULAR CASE WE WILL THE FRACTIONAL Q-DIFFERENTIATION OF THE GENERALIZED M-SERIES AND EXPONENTIAL SERIES.

THEOREM 1: IF THE Robotnov and Hartley Function $F_\alpha[a,z]$ converges absolutely for $|q| < \boldsymbol{\rho}$ **THEN**

$$D_{z,q}^{\mu}\left\{z^{\lambda-1}\sum_{k=0}^{\infty}\frac{(a)^k z^{(k+1)\alpha-1}}{\Gamma(k\alpha+\alpha)}\right\}=\sum_{k=0}^{\infty}\frac{(a)^k}{\Gamma(k\alpha+\alpha)}D_{z,q}^{\mu}z^{(k+1)\alpha+\lambda-2} \tag{2.1}$$

Where **RE** $(\lambda) > \mathbf{0}$, **RE** $(\mu) < \mathbf{0}$, $\mathbf{0} < |q| < \mathbf{1}$

PROOF: STARTING FROM THE LEFT SIDE AND USING EQUATION (2.1), WE HAVE

$$D_{z,q}^{\mu}\left\{z^{\lambda-1}\sum_{k=0}^{\infty}\frac{(a)^k z^{(k+1)\alpha-1}}{\Gamma(k\alpha+\alpha)}\right\}=\frac{1}{\Gamma_Q(-\mu)}\int_0^z (z-yq)_{-\mu-1}\,y^{\lambda-1}\sum_{k=0}^{\infty}\frac{a^k y^{(k+1)\alpha-1}}{\Gamma(k\alpha+\alpha)}d(y;q)$$

$$=\frac{z^{\lambda-\mu-1}}{\Gamma_Q(-\mu)}\int_0^1 (1-tq)_{-\mu-1}\,t^{\lambda-1}\sum_{k=0}^{\infty}\frac{a^k z^{(k+1)\alpha-1}t^{(k+1)\alpha-1}}{\Gamma(k\alpha+\alpha)}d(t;q) \tag{2.2}$$

NOW THE FOLLOWING OBSERVATION ARE MADE

(i) $\sum_{k=0}^{\infty}\frac{a^k z^{(k+1)\alpha-1}t^{(k+1)\alpha-1}}{\Gamma(k\alpha+\alpha)}$ converges absolutely and therefore uniformly on domain of x over the region of integration.

(ii) $\int_0^1\left|(1-tq)_{-\mu-1}t^{\lambda-1}\right|\,d(t;q)$ **IS CONVERGENT,**

PROVIDED **RE** $(\lambda) > \mathbf{0}$, **RE** $(\mu) < \mathbf{0}$, $\mathbf{0} < |q| < \mathbf{1}$

THEREFORE THE ORDER OF INTEGRATION AND SUMMATION CAN BE INTERCHANGED IN (2.2) TO OBTAIN.

$$=\frac{z^{\lambda-\mu-1}}{\Gamma_Q(-\mu)}\sum_{k=0}^{\infty}\frac{a^k z^{(k+1)\alpha-1}}{\Gamma(k\alpha+\alpha)}\int_0^1 (1-tq)_{-\mu-1}\,t^{(k+1)\alpha+\lambda-2}d(t;q)$$

$$=\sum_{k=0}^{\infty}\frac{a^k}{\Gamma(k\alpha+\alpha)}D_{z,q}^{\mu}z^{(k+1)\alpha+\lambda-2}$$

Hence the statement (5) is proved.

3. Some special cases:

(i) If we take $\alpha = 1$ in equation (2.1) it becomes the fractional q-derivative of power series.

$$D_{z,q}^{\mu}\left\{z^{\lambda-1}\sum_{k=0}^{\infty}\frac{(a)^k z^k}{\Gamma(k+1)}\right\}=\sum_{k=0}^{\infty}\frac{(a)^k}{\Gamma(k+1)}D_{z,q}^{\mu}\left\{z^{k+\lambda-1}\right\} \tag{3.1}$$

This equation (3.1) is known result given by Yadav and Purohit [8] and Ali, Jain and Sharma [9].

(ii) When $\alpha = 1$ and $a = 1$ in(2.1), we have

$$D_{z,q}^{\mu}\left\{z^{\lambda-1}\sum_{k=o}^{\infty}\frac{z^k}{\Gamma(k+1)}\right\}=\sum_{k=o}^{\infty}\frac{1}{k!}D_{z,q}^{\mu}\left\{z^{k+\lambda-1}\right\} \tag{3.2}$$

EQUIVALENTLY,

$$D_{z,q}^{\mu}\left\{z^{\lambda-1}e^{z}\right\}=\sum_{k=o}^{\infty}\frac{1}{k!}D_{z,q}^{\mu}\left\{z^{k+\lambda-1}\right\} \tag{3.3}$$

Thus the equation reduces to fractional q-derivative of exponential function.

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