ISSN 2348 – 0319 International Journal of Innovative and Applied Research (2014), Volume 2, Issue (5): 55 - 64

Journal home page: http://www.journalijiar.com

INTERNATIONAL JOURNAL OF INNOVATIVE AND APPLIED RESEARCH

ISSN 2348 - 0319

RESEARCH ARTICLE

On Fuzzy γ - Pre Open Sets and Fuzzy γ - Pre Closed Sets in Fuzzy Topological Spaces

K. Bageerathi

Department of Mathematics, Aditanar College of Arts and Science, Tiruchendur-628216, India.

Abstract:

The aim of this paper is to introduce the concept of fuzzy γ- pre open and fuzzy γ- pre closed sets of a fuzzy topological space. Some characterizations are discussed, examples are given and properties are established. Also, we define fuzzy γ- pre interior and fuzzy γ- pre closure operators. And we introduce fuzzy γ- t-set, γ-PO extremely disconnected space analyse them.

MSC 2010: 54A40, 03E72.

Key Words: Fuzzy γ - open, fuzzy γ - closed, fuzzy γ- pre open, fuzzy γ- pre closed, fuzzy γ- pre interior and fuzzy γ- pre closure, γ- t-set and fuzzy topology.

Introduction

The concepts of fuzzy sets and fuzzy set operations were first introduced by L.A.Zadeh [10] in his paper. Let X be a non empty set and I be the unit interval [0,1]. A Fuzzy set in X is a mapping from X in to I. In 1968, Chang [3] introduced the concept of fuzzy topological space which is a natural generalization of topological spaces. Our notation and terminology follow that of Chang. Singal[8] introduced the notions of fuzzy pre open and fuzzy pre closed sets. And T.Noiri and O.R.Sayed[5] introduced the notion of γ-open sets and γ-closed sets. Swidi Oon[4] studied some of its properties.

Through this paper (X, τ) (or simply X), denote fuzzy topological spaces. For a fuzzy set A in a fuzzy topological space X, cl(A), int(A), A^C denote the closure, interior, complement of A respectively. By 0_x and l_x we mean the constant fuzzy sets taking on the values 0 and 1, respectively.

In this paper we introduce fuzzy γ-pre open sets and fuzzy γ-pre closed sets its properties are established in fuzzy topological spaces. The concepts that are needed in this paper are discussed in the second section. The concepts of fuzzy γ-pre open and fuzzy γ-pre closed sets in fuzzy topological spaces and studied their properties in the third and fourth section respectively. Using the fuzzy γ- pre open sets, we introduce the concept of fuzzy γ-PO extremely disconnected space. The section 5 and 6 are dealt with the concepts of fuzzy γ-pre interior and γ-pre closure operators. In the last section, we define fuzzy γ-t-sets and discuss the relations between this set and the sets defined previously.

2. Preliminaries

In this section, we give some basic notions and results that are used in the sequel.

Definition 2.1: A fuzzy set A of a fuzzy topological space X is called:

1) fuzzy pre open (pre closed) [2] if $A \leq int(clA))(A \geq cl(int(A))$.
2) fuzzy strongly pre open (strongly pre closed) [4] if $A \leq int(pcl(A))$ $(A \geq cl(pint(A))$.
3) fuzzy γ-open (fuzzy γ-closed) [5] if $A \leq (int(cl A)) \vee cl(int(A))$ $(A \geq (cl(int(A))) \wedge (int (cl(A))))$.

4) fuzzy strongly semi open (strongly semi closed) if $A \leq int(cl(int(A)))(A \geq cl(int(cl(A)))$.

Definition 2.2[7]: If λ is a fuzzy set of X and μ is a fuzzy set of Y, then $(\lambda \times \mu)(x, y)= \min \{ \lambda(x), \mu(y)\}$, for each X×Y.

Definition 2.3[2]: An fuzzy topological space (X, τ_1) is a product related to an fuzzy topological space (Y, τ_2) if for fuzzy sets A of X and B of Y whenever $C^c \not\geq A$ and $D^c \not\geq B$ implies $C^c \times 1 \vee 1 \times D^c \geq A \times B$, where $C \in \tau_1$ and $D \in \tau_2$, there exist $C_1 \in \tau_1$ and $D_1 \in \tau_2$ such that $C_1^c \geq A$ or $D_1^c \geq B$ and $C_1^c \times 1 \vee 1 \times D_1^c = C^c \times 1 \vee 1 \times D^c$.

Lemma 2.4 [2]: Let X and Y be fuzzy topological spaces such that X is product related to Y. Then for fuzzy sets A of X and B of Y,

1) $cl(A \times B) = cl(A) \times cl(B)$
2) $int(A \times B) = int(A) \times int(B)$

Lemma 2.5[1]: For fuzzy sets A, B, C and D in a set S, one has
$(A \wedge B) \times (C \wedge D) = (A \times D) \wedge (B \times C)$

Remark 2.6[5]:
1. Any union of fuzzy γ-open sets in a fuzzy topological space X is a fuzzy γ-open set.
2. Any intersection of fuzzy γ-closed sets is fuzzy γ-closed set.
3. Let $\{A_\alpha\}_{\alpha\in\Delta}$ be a collection of fuzzy γ-open sets in a fuzzy topological space X. Then $\vee_{\alpha\in\Delta} A_\alpha$ is fuzzy γ-open.

Definition 2.7[5]: Let A be any fuzzy set in the fuzzy topological space X. Then we define
$\gamma\text{-cl}(A) = \wedge\{B : B \geq A, B \text{ is fuzzy } \gamma\text{-closed}\}$ and
$\gamma\text{-int}(A) = \vee\{B : B \leq A, B \text{ is fuzzy } \gamma\text{-open in } X\}$.
Properties 2.8[5]: Let A be any fuzzy set in the fuzzy topological space X. Then

a) $\gamma\text{-cl}(A^c) = (\gamma\text{-int}(A))^c$
b) $\gamma\text{-int}(A^c) = (\gamma\text{-cl}(A))^c$

Properties 2.9[5]: Let A and B be any two fuzzy sets in a fuzzy topological space X. Then
1) $\gamma\text{-int}(0) = 0$, $\gamma\text{-int}(1) = 1$.
2) γ-int (A) is fuzzy γ-open in X.
3) $\gamma\text{-int}(\gamma\text{-int}(A)) = \gamma\text{-int}(A)$.
4) if $A \leq B$ then $\gamma\text{-int}(A) \leq \gamma\text{-int}(B)$.
5) $\gamma\text{-int}(A\wedge B) = \gamma\text{-int}(A) \wedge \gamma\text{-int}(B)$.
6) $\gamma\text{-int}(A\vee B) \geq \gamma\text{-int}(A) \vee \gamma\text{-int}(B)$.

Properties 2.10[5]: Let A and B be any two fuzzy sets in a fuzzy topological space X. Then
1) $\gamma\text{-cl}(0) = 0$, $\gamma\text{-cl}(1) = 1$.
2) γ-cl(A) is fuzzy γ-closed in X.
3) $\gamma\text{-cl}(\gamma\text{-cl}(A)) = \gamma\text{-cl}(A)$.
4) if $A \leq B$ then $\gamma\text{-cl}(A) \leq \gamma\text{-cl}(B)$.
5) $\gamma\text{-cl}(A\vee B) = \gamma\text{-cl}(A) \vee \gamma\text{-cl}(B)$.
6) $\gamma\text{-cl}(A\wedge B) \leq \gamma\text{-cl}(A) \wedge \gamma\text{-cl}(B)$.

3. Fuzzy γ-Pre Open Sets

In this section we introduce the concept of fuzzy γ-pre open sets in a fuzzy topological space.

Definition 3.1: Let A be a fuzzy subset of a fuzzy topological space (X,τ). Then A is called fuzzy γ-pre open if $A \leq \gamma\text{-int}(cl(A))$.

Remarks 3.2: It is obvious that every fuzzy γ-open is fuzzy γ-pre open and every fuzzy open set is fuzzy γ-pre open but the separate converses may not be true as shown by the following example.
The following example shows that every fuzzy γ-pre open set need not be fuzzy γ-open.

Example 3.3: Let X = {a, b, c} and $\tau = \{0, 1, \{a_{.2}, b_{.3}, c_{.5}\}, \{a_{.4}, b_{.7}, c_{.3}\}, \{a_{.2}, b_{.3}, c_{.3}\}, \{a_{.4}, b_{.7}, c_{.5}\}\}$. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is $\tau^c = \{0, 1, \{a_{.8}, b_{.7}, c_{.5}\}, \{a_{.6}, b_{.3}, c_{.7}\}, \{a_{.8}, b_{.7}, c_{.7}\}, \{a_{.6}, b_{.3}, c_{.5}\}\}$. Let $A = \{a_{.4}, b_{.6}, c_{.6}\}$. Then $cl(int(A)) = \{a_{.6}, b_{.3}, c_{.5}\}$ and $int(cl(A)) = \{a_{.4}, b_{.7}, c_{.5}\}$. Therefore $int(cl(A)) \vee cl(int(A)) = \{a_{.6}, b_{.7}, c_{.5}\}$. By Definition 2.1(3), A is not fuzzy γ-open. Now $\gamma\text{-int}(cl(A)) = \{a_{.6}, b_{.7}, c_{.7}\}$. Then $A \leq \gamma\text{-int}(cl(A))$. Therefore A is fuzzy γ-pre open.

The next example shows that every fuzzy γ-pre open set need not be fuzzy open.

Example 3.4: Let X = {a, b} and τ = {0, 1,{$a_{.2}$, $b_{.2}$}}. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is τ^c = {0, 1, {$a_{.8}$, $b_{.8}$}}. Let A = {$a_{.3}$, $b_{.4}$}. Then γ-int(cl(A)) = {$a_{.6}$, $b_{.6}$}. It shows that A ≤ γ-int (cl(A)). By using Definition 3.1, A is fuzzy γ-pre open. But A is not a fuzzy open set.

It is clear that every fuzzy pre open is fuzzy γ-pre open. Every fuzzy strongly semi open set and fuzzy strongly pre open set is also fuzzy γ-pre open. But the converse need not be true as shown by the following example. The following example shows that every fuzzy γ-pre open set need not be fuzzy pre open.

Example 3.5: Let X = {a, b} and τ = {0, 1, {$a_{.2}$, $b_{.3}$}}. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is τ^c = {0, 1, {$a_{.8}$, $b_{.7}$}}. Let A = {$a_{.2}$, $b_{.4}$}. Then γ-int(cl(A)) = {$a_{.8}$, $b_{.6}$}. It shows that A ≤ γ-int (cl(A)). By using Definition3.1, A is fuzzy γ-pre open. Now int(cl(A)) = { $a_{.2}$, $b_{.3}$}. That shows A $\nleq$ int (cl(A)) . Hence A is not a fuzzy pre open set.

The next example shows that every fuzzy γ-pre open need not be fuzzy strongly pre open.

Example 3.6: Let X = {a, b} and τ = {0, 1, {$a_{.3}$, $b_{.2}$}, {$a_{.7}$, $b_{.6}$}}. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is τ^c = {0, 1, {$a_{.7}$, $b_{.8}$}, {$a_{.3}$, $b_{.4}$}}. Let A = {$a_{.3}$, $b_{.3}$}. Then pcl(A)= {$a_{.4}$, $b_{.4}$} and int(pcl(A))= {$a_{.3}$, $b_{.2}$}. That shows A $\nleq$ int(pcl(A)), it follows that A is not fuzzy strongly pre open. But calculations give γ-int(cl(A)) = {$a_{.3}$, $b_{.4}$}. That is A≤ γ-int(cl(A)). Therefore A is fuzzy γ-pre open.

The next example shows that every fuzzy γ-pre open set need not be fuzzy strongly semi open.

Example 3.7: Let X = {a, b} and τ = {0, 1, {$a_{.3}$, $b_{.2}$}, {$a_{.3}$, $b_{.5}$}}. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is τ^c = {0, 1, {$a_{.7}$, $b_{.8}$}, {$a_{.7}$, $b_{.5}$}}. Let A = {$a_{.3}$, $b_{.4}$}. Then γ-int(cl(A))= {$a_{.4}$, $b_{.5}$}. That shows A ≤ γ-int(cl(A)), it follows that A is fuzzy γ-pre open. But int(cl(int(A))) = {$a_{.3}$, $b_{.5}$}, that is A $\nleq$ int(cl(int(A)). Therefore A is not fuzzy strongly semi open.

Proposition 3.8: Let (X, τ) be a fuzzy topological space. Then the union of any two fuzzy γ-pre open sets is a fuzzy γ-pre open set.

Proof: Let A_1 and A_2 be the two fuzzy γ-pre open sets. By Definition 3.1, $A_1 \leq$ γ-int (cl(A_1)) and $A_2 \leq$ γ-int (cl(A_2)). Therefore $A_1 \vee A_2 \leq$ γ-int (cl(A_1)) ∨ γ-int(cl(A_2)). By using Properties 2.9(6), $A_1 \vee A_2 \leq$ γ-int (cl(A_1)∨cl(A_2)) = γ-int (cl($A_1 \vee A_2$)). Hence $A_1 \vee A_2$ is fuzzy γ-pre open.

The following example shows that the intersection of any two fuzzy γ-pre open sets need not be fuzzy γ-pre open set.

Example 3.9: Let X = {a, b} and τ = {0, 1, {$a_{.2}$, $b_{.3}$}, {$a_{.3}$, $b_{.4}$}}. Then (X, τ) be a fuzzy topological space. The family of all fuzzy closed sets of τ is τ^c = {0, 1, {$a_{.8}$, $b_{.7}$}, {$a_{.7}$, $b_{.6}$}}. Let A = {$a_{.8}$, $b_{.9}$}. Then γ-int (cl(A))= {1}. Thus by Definition 3.1, A is fuzzy γ-pre open. Let B = {$a_{.9}$, $b_{.7}$}. Then γ-int(cl(B))= {1}.Thus by Definition 3.1, B is fuzzy γ-pre open. Now A ∧ B = {$a_{.8}$, $b_{.7}$} and γ-int (cl(A ∧ B)) = {$a_{.7}$, $b_{.4}$}. Thus A ∧ B $\nleq$ γ-int (cl(A ∧ B)). Therefore A ∧ B is not fuzzy γ-pre open.

Theorem 3.10: Let (X, τ) be a fuzzy topological space and let $\{A_\alpha\}_{\alpha\in\Delta}$ be a collection of fuzzy γ-pre open sets in a fuzzy topological space X. Then $\vee_{\alpha\in\Delta} A_\alpha$ is fuzzy γ-pre open.

Proof: Let Δ be a collection of fuzzy γ-pre open sets of a fuzzy topological space (X, τ). Then by using Definition 3.1, for each α∈Δ, $A_\alpha \leq$ γ-int(cl(A_α)). Thus $\vee_{\alpha\in\Delta} A_\alpha \leq \vee_{\alpha\in\Delta}$ γ-int(cl(A_α)). By using Remark 2.6(3), $\vee_{\alpha\in\Delta} A_\alpha \leq$ γ-int ($\vee_{\alpha\in\Delta}$ (cl(A_α)). Since ∨cl(A_α) ≤ cl(∨A_α), $\vee_{\alpha\in\Delta} A_\alpha \leq$ γ-int (cl$(\vee_{\alpha\in\Delta} A_\alpha)$). Thus the arbitrary union of fuzzy γ-pre open sets is fuzzy γ-pre open.

Lemma 3.11[9]: Let (X, τ) and (Y, σ) be any two fuzzy topological spaces such that X is product related to Y. Then the product $A_1 \times A_2$ of fuzzy γ-open set A_1 of X and a fuzzy γ-open set A_2 of Y is fuzzy γ-open set of the fuzzy product space X × Y.

Theorem 3.12: Let (X, τ) and (Y, σ) be any two fuzzy topological spaces such that X is product related to Y. Then the product $A_1 \times A_2$ of a fuzzy γ-pre open set A_1 of X and a fuzzy γ-pre open set A_2 of Y is fuzzy γ-pre open set of the fuzzy product space $X \times Y$.

Proof: Let A_1 be a fuzzy γ-pre open subset of X and A_2 be a fuzzy γ-pre open subset of Y. Then by using Definition 3.1, we have $A_1 \leq \gamma\text{-int}(cl(A_1))$ and $A_2 \leq \gamma\text{-int}(cl(A_2))$. This implies that, $A_1 \times A_2 \leq \gamma\text{-int}(cl(A_1)) \times \gamma\text{-int}(cl(A_2))$. By Lemma 3.11, $A_1 \times A_2 \leq \gamma\text{-int}(cl(A_1) \times cl(A_2))$. By Lemma 2.4(1), $A_1 \times A_2 \leq \gamma\text{-int}(cl(A_1 \times A_2))$.Therefore $A_1 \times A_2$ is fuzzy γ-pre open set in the fuzzy product space $X \times Y$.

4. Fuzzy γ – pre closed sets

In this section we introduce the concept of fuzzy γ-pre closed sets in a fuzzy topological space.

Definition 4.1: Let A be a fuzzy subset of a fuzzy topological space (X, τ). Then A is called fuzzy γ-pre closed set of X if $A \geq \gamma\text{-cl}(int(A))$.

Proposition 4.2: Let (X, τ) be a fuzzy topological space and A be a fuzzy subset of X. Then A is fuzzy γ-pre closed if and only if A^c is fuzzy γ-pre open.

Proof : Let A be a fuzzy γ-pre closed subset of X. Then by Definition 4.1, $A \geq \gamma\text{-cl}(int(A))$. Taking complement on both sides, we get $A^c \leq (\gamma\text{-cl}(int(A)))^c$. By using Properties 2.8(b), $A^c \leq \gamma\text{-int}(cl(A^c))$. By Definition 4.1, we have A^c is fuzzy γ-pre open.

Conversely, let A^c is fuzzy γ-pre open. By Definition 3.1, $A^c \leq \gamma\text{-int}(cl(A^c))$.Taking complement on both sides we get, $A \geq (\gamma\text{-int}(cl(A^c)))^c$. By using Properties 2.8(a), $A \geq \gamma\text{-cl}(int(A))$. By Definition 4.1, we have A is fuzzy γ-pre closed.

Remark 4.3: It is obvious that every fuzzy γ-closed set is fuzzy γ-pre closed and every fuzzy closed set is fuzzy γ-pre closed. Every fuzzy strongly semi closed set and fuzzy strongly pre closed set is also fuzzy γ-pre closed. But the separate converses may not be true as shown by the following example.

The following example shows that every fuzzy γ-pre closed set need not be fuzzy γ-closed.

Example 4.4: Let $X = \{a, b, c\}$ and $\tau = \{0, 1, \{a_{.2}, b_{.3}, c_{.5}\}, \{a_{.4}, b_{.7}, c_{.3}\}, \{a_{.2}, b_{.3}, c_{.3}\}, \{a_{.4}, b_{.7}, c_{.5}\}\}$. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is $\tau^c = \{0, 1, \{a_{.8}, b_{.7}, c_{.5}\}, \{a_{.6}, b_{.3}, c_{.7}\}, \{a_{.8}, b_{.7}, c_{.7}\},\{a_{.6}, b_{.3}, c_{.5}\}\}$. Let $A = \{a_{.3}, b_{.6}, c_{.5}\}$ then $cl(int(A))= \{a_{.6}, b_{.3}, c_{.5}\}$ and $int(cl(A))= \{a_{.4}, b_{.7}, c_{.5}\}$. Then $int(cl(A)) \wedge cl(int(A)) = \{a_{.4}, b_{.3}, c_{.5}\}$. By Definition 2.1(3), A is not fuzzy γ-closed. Now let $\gamma\text{-cl}(int(A) = \{a_{.3}, b_{.3}, c_{.5}\}$. Then $A \geq \gamma\text{-cl}(int(A))$. Thus A is fuzzy γ-pre closed.

The next example shows that every fuzzy γ-pre closed need not be fuzzy closed.

Example 4.5: Let $X = \{a, b\}$ and $\tau = \{0, 1, \{a_{.3}, b_{.4}\}\}$. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is $\tau^c = \{0, 1, \{a_{.7}, b_{.6}\}\}$. Let $A = \{a_{.4}, b_{.5}\}$. Then $\gamma\text{-cl}(int(A))= \{a_{.4}, b_{.4}\}$. That shows $A \geq \gamma\text{-cl}(int(A))$, it follows that A is fuzzy γ-pre closed. But A is not a fuzzy closed set.

The next example shows that every fuzzy γ-pre closed and fuzzy strongly semi closed need not be fuzzy strongly pre closed.

Example 4.6: Let $X = \{a, b\}$ and $\tau = \{0, 1, \{a_{.3}, b_{.2}\}, \{a_{.7}, b_{.6}\}\}$. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is $\tau^c = \{0, 1, \{a_{.7}, b_{.8}\}, \{a_{.3}, b_{.4}\}\}$. Let $A = \{a_{.5}, b_{.6}\}$. Then $pint(A))= \{a_{.5}, b_{.5}\}$ and $cl(pint(A))= \{a_{.7}, b_{.8}\}$. That shows $A \ngeq cl(pint(A))$, it follows that A is not fuzzy strongly pre closed. But $A \geq \gamma\text{-cl}(int(A)) = \{a_{.3}, b_{.3}\}$. This implies that A is fuzzy γ-pre closed. Now $cl(int(cl(A)))= \{a_{.7}, b_{.8}\}$. It follows that $A \ngeq cl(int(cl(A)))$. Therefore A is not fuzzy strongly semi closed.

It follows that every fuzzy pre closed set is fuzzy γ-pre closed but the converse may not be true as shown by the following example.

Example 4.7: Let $X = \{a, b\}$ and $\tau = \{0, 1, \{a_{.2}, b_{.3}\}\}$. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is $\tau^c = \{0, 1, \{a_{.8}, b_{.7}\}\}$. Let $A = \{a_{.4}, b_{.5}\}$. Then $\gamma\text{-cl}(int(A)) = \{a_{.3}, b_{.5}\}$. It shows that $A \geq \gamma\text{-cl}(int(A))$. By using Definition 4.1, A is fuzzy γ-pre closed. Now $cl(int(A)) = \{ a_{.8}, b_{.7}\}$. That shows $A \ngeq cl(int(A))$. Hence A is not a fuzzy pre closed set.

Theorem 4.8: Let (X, τ) be a fuzzy topological space. Then the intersection of two fuzzy γ-pre closed sets is fuzzy γ-pre closed set in the fuzzy topological space (X, τ).

Proof: Let A_1 and A_2 be two fuzzy γ-pre closed sets. By Definition 4.1, we have $A_1 \geq \gamma\text{-cl}(\text{int}(A_1))$ and $A_2 \geq \gamma\text{-cl}(\text{int}(A_2))$. By using Properties 2.10(6), $A_1 \wedge A_2 \geq \gamma\text{-cl}(\text{int}(A_1) \wedge \text{int}(A_2)) = \gamma\text{-cl}(\text{int}(A_1 \wedge A_2))$. Hence $A_1 \wedge A_2$ is fuzzy γ-pre closed.

The union of two fuzzy γ-pre closed sets is need not be fuzzy γ-pre closed set in the fuzzy topological space X as shown by the following example.

Example 4.9: Let X = {a, b} and τ = {0, 1, {$a_{.5}$, $b_{.5}$}, {$a_{.4}$, $b_{.5}$}}. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is τ^c = {0, 1, {$a_{.5}$, $b_{.5}$}, {$a_{.6}$, $b_{.5}$}}. Let A = {a_0, $b_{.5}$}. Then γ-cl(int(A)) = {0}.Thus by Definition 4.1, A is fuzzy γ-pre closed. Let B = {$a_{.4}$, b_0}. Then we get γ-cl(int(B))= {0}. Thus by Definition 4.1, B is fuzzy γ-pre closed. Now $A \vee B$ = {$a_{.4}$, $b_{.5}$} and γ-cl (int($A \vee B$)) = {$a_{.5}$, $b_{.5}$}. Thus $A \vee B \ngeq \gamma\text{-cl}(\text{int}(A \vee B))$. Therefore $A \vee B$ is not fuzzy γ-pre closed.

Theorem 4.10: Let (X, τ) be a fuzzy topological space and let $\{A_\alpha\}_{\alpha\in\Delta}$ be a collection of fuzzy γ-pre closed sets in a fuzzy topological space X. Then $\wedge_{\alpha\in\Delta} A_\alpha$ is fuzzy γ-pre closed for each α∈Δ.

Proof: Let Δ be a collection of fuzzy γ-pre closed sets of a fuzzy topological space (X, τ). Then by Definition 4.1, for each α∈Δ, $A_\alpha \geq \gamma\text{-cl}(\text{int}(A_\alpha))$. Then $\wedge_{\alpha\in\Delta} A_\alpha \geq \wedge_{\alpha\in\Delta}(\gamma-\text{cl}(\text{int}(A_\alpha)))$. By using Properties 2.6, $\wedge_{\alpha\in\Delta} A_\alpha \geq \gamma\text{-cl}(\text{int}(\wedge_{\alpha\in\Delta}(A_\alpha)))$. Thus arbitrary intersection of fuzzy γ-pre closed set is fuzzy γ-pre closed.

Lemma 4.11[9]: Let (X, τ) and (Y, σ) be any two fuzzy topological spaces such that X is product related to Y. Then the product $A_1 \times A_2$ of fuzzy γ-closed set A_1 of X and a fuzzy γ-closed set A_2 of Y is fuzzy γ-closed set of the fuzzy product space X × Y.

Theorem 4.12: Let (X, τ) and (Y, σ) be any two fuzzy topological spaces such that X is product related to Y. Then the product $A_1 \times A_2$ of fuzzy γ-pre closed set A_1 of X and a fuzzy γ-pre closed set A_2 of Y is fuzzy γ-pre closed set of the fuzzy product space X × Y.

Proof: Let A_1 be a fuzzy γ-pre closed subset of X and A_2 be a fuzzy γ-pre closed subset of Y. Then by Definition 4.1, we have $A_1 \geq \gamma\text{-cl}(\text{int}(A_1))$ and $A_2 \geq \gamma\text{-cl}(\text{int}(A_2))$. Now $A_1 \times A_2 \geq (\gamma\text{-cl}(\text{int}(A_1)) \times \gamma\text{-cl}(\text{int}(A_2))$. By using Lemma 4.11, $A_1 \times A_2 \geq \gamma\text{-cl}(\text{int}(A_1) \times \text{int}(A_2))$. By using the Lemma 2.4(2), we get $A_1 \times A_2 \geq \gamma\text{-cl}(\text{int}(A1 \times A2))$. Therefore $A_1 \times A_2$ is fuzzy γ-pre closed in the fuzzy product space X × Y.

5. Fuzzy γ-pre interior

In this section we introduce the concept of fuzzy γ-pre interior and their properties in a fuzzy topological space.

Definition 5.1: Let (X, τ) be a fuzzy topological space. Then for a fuzzy subset A of X, the fuzzy γ-pre interior of A (briefly γ-pint (A)) is the union of all fuzzy γ-pre open sets of X contained in A. That is, γ-pint (A) = ∨ {B: B ≤ A, B is fuzzy γ-pre open in X}

Proposition 5.2 : Let (X, τ) be a fuzzy topological space. Then for any fuzzy subsets A and B of a fuzzy topological X we have

(i) γ-pint (A) ≤ A,
(ii) A is fuzzy γ-pre open ⇔ γ-pint(A) = A,
(iii) γ-pint(γ-pint (A)) = γ-pint (A),
(iv) if A ≤ B then γ-pint (A) ≤ γ-pint (B).

Proof:

(i) follows from Definition 5.1. Let A be fuzzy γ-pre open. Then A ≤ γ-pint (A). By using (i) we get A = γ-pint (A). Conversely assume that A = γ-pint (A). By using Definition 5.1, A is fuzzy γ-pre open. Thus (ii) is proved. By using (ii) we get γ-pint(γ-pint (A)) = γ-pint (A) . This proves (iii). Since A ≤ B, by using (i) γ-pint (A)≤ A ≤ B. That is γ-pint (A)≤ B. By (iii), γ-pint (γ-pint (A))≤ γ-pint(B). Thus γ-pint (A) ≤ γ-pint (B). This proves (iv).

Theorem 5.3: Let (X, τ) be a fuzzy topological space. Then for any fuzzy subset A and B of a fuzzy topological space, we have

(i) γ-pint (A∧B) = (γ-pint A) ∧ (γ-pint B)
(ii) γ-pint (A∨B) ≥ (γ-pint A) ∨ (γ-pint B)

Proof: Since A∧B ≤ A and A∧B ≤ B, by using Proposition 5.2(iv), we get γ-pint (A∧B) ≤ γ-pint(A) and γ-pint (A∧B) ≤ γ-pint (B). This implies that γ-pint (A ∧ B) ≤ (γ-pint A) ∧ (γ-pint B) --- (1).
By using Proposition5.2(i), we have γ-pint(A) ≤ A and γ-pint(B) ≤ B. This implies that γ-pint (A) ∧ γ-pint(B) ≤ A ∧ B. Now applying Proposition 5.2(iv),
we get γ-pint ((γ-pint (A) ∧ γ-pint (B)) ≤ γ-pint (A∧B).
By (1), γ-pint(γ-pint (A)) ∧ γ-pint (γ-pint (B)) ≤ γ-pint (A ∧ B). By Proposition 5.2(iii), γ-pint (A) ∧ γ-pint (B) ≤ γ-pint (A ∧ B) -----------------------(2). From (1) and (2),
γ-pint (A ∧ B) = γ-pint(A) ∧ γ-pint (B). This implies(i).
Since A ≤ A∨ B and B≤ A∨B, by using Proposition 5.2(iv), we have γ-pint (A) ≤ γ-pint (A ∨ B) and γ-pint (B) ≤ γ-pint (A ∨ B). This implies that γ-pint (A) ∨ γ-pint (B) ≤ γ-pint (A ∨ B). Hence (ii).

The following example shows that the equality need not be hold in Theorem 5.3(ii).

Example 5.4: Let X = {a, b, c} and τ = {0, 1, {$a_{.2}$, $b_{.3}$, $c_{.5}$}, {$a_{.4}$, $b_{.7}$, $c_{.3}$}, {$a_{.2}$, $b_{.3}$, $c_{.3}$}, {$a_{.4}$, $b_{.7}$, $c_{.5}$}}. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is τ^c = {0, 1, {$a_{.8}$, $b_{.7}$, $c_{.5}$}, {$a_{.6}$, $b_{.3}$, $c_{.7}$}, {$a_{.8}$, $b_{.7}$, $c_{.7}$}, {$a_{.6}$, $b_{.3}$, $c_{.5}$}}. Consider A = {$a_{.3}$, $b_{.5}$, $c_{.3}$} and B = {$a_{.7}$, $b_{.4}$, $c_{.6}$}. Then γ-pint (A) = {$a_{.3}$, $b_{.3}$, $c_{.3}$} and γ-pint (B) = {$a_{.5}$, $b_{.4}$, $c_{.5}$}. That implies γ-pint (A) ∨ γ-pint (B) = {$a_{.5}$, $b_{.4}$, $c_{.5}$}. Now A∨B = {$a_{.7}$, $b_{.5}$, $c_{.6}$}, it follows that γ-pint(A∨B) = {$a_{.5}$, $b_{.5}$, $c_{.5}$}. Then γ-pint (A∨B)≰ γ-pint (A) ∨ γ-pint (B). Thus γ-pint (A∨B) ≠ γ-pint (A) ∨ γ-pint (B).

6. Fuzzy γ-pre closure

In this section we introduce the concept of fuzzy γ-pre closure in a fuzzy topological space.

Definition 6.1: Let (X, τ) be a fuzzy topological space. Then for a fuzzy subset A of X, the fuzzy γ-pre closure of A (briefly γ-pcl (A)) is the intersection of all fuzzy γ-pre closed sets contained in A. That is, γ-pcl (A) = ∧{ B: B ≥ A, B is fuzzy γ-pre closed}.

Proposition 6.2: Let (X, τ) be a fuzzy topological space. Then for any fuzzy subsets A of X, we have i. $(\gamma\text{-pint } (A))^c$ = γ-pcl (A^c) and ii. $(\gamma\text{-pcl } (A))^c$ = γ-pint (A^c)

Proof: By using Definition 5.1, γ-pint (A) = ∨{B: B≤ A, B is fuzzy γ-pre open}. Taking complement on both sides, we get $[\gamma\text{-pint } (A)]^c$ = (sup{B : B ≤ A, B is fuzzy γ-pre open}$)^c$ = inf{B^c : $B^c \geq A^c$, B^c is fuzzy γ-pre closed }. Replacing B^c by c, we get $[\gamma\text{-pint } (A)]^c$ = ∧{c: c ≥ A^c, c is fuzzy γ-pre closed }. By Definition 6.1, $[\gamma\text{-pint } (A)]^c$ = γ-pcl(A^c). This proves(i). By using(i), $[\gamma\text{-pint } (A^c)]^c$ = γ-pcl $(A^c)^c$ = γ-pcl (A). Taking complement on both sides, we get γ-pint (A^c)= $[\gamma\text{-pcl } (A)]^c$. Hence proved (ii).

Proposition 6.3: Let (X, τ) be a fuzzy topological space. Then for a fuzzy subset A and B of a fuzzy topological space X, we have

(i) A ≤ γ-pcl (A).
(ii) A is fuzzy γ-pre closed ⇔ γ-pcl (A) = A.
(iii) γ-pcl (γ-pcl (A)) = γ-pcl (A).
(iv) if A ≤ B then γ-pcl (A) ≤ γ-pcl (B).

Proof:

(i) The proof of (i) follows from the Definition 6.1.
(ii) Let A be fuzzy γ-pre closed subset in X. By using Proposition 4.2, A^c is fuzzy γ-pre open. By using Proposition 6.2(ii), γ-pint(A^c) = A^c ⇔ $[\gamma\text{-pcl } (A)]^c$ = A^c ⇔ γ-pcl (A) = A. Thus proved (ii).

(iii) By using (ii), γ-pcl(γ-pcl (A)) = γ-pcl (A). This proves (iii).

(iv) Suppose $A \leq B$. Then $B^c \leq A^c$. By using Proposition 5.2(iv), γ-pint (B^c)≤ γ-pint (A^c). Taking complement on both sides, we get $[\gamma\text{-pint }(B^c)]^c \geq [\gamma\text{-pint }(A^c)]^c$. By proposition 6.2(ii), γ-pcl (B) ≥ γ-pcl(A). This proves (iv).

Proposition 6.4: Let A be a fuzzy set in a fuzzy topological space X. Then int(A)≤ pint(A) ≤ γ-int(A) ≤ γ-pint(A) ≤ A ≤ γ-pcl(A) ≤ γ-cl(A) ≤ pcl(A) ≤ cl(A).

Proof: It follows from the Definitions of corresponding operators.

Theorem 6.5: 1. A fuzzy set B of an fuzzy topological space X is fuzzy γ-pre open if and only if there exists a fuzzy set A of X such that A≤B≤ γ-int(cl(A)).

1. A fuzzy set B of an fuzzy topological space X is fuzzy γ-pre closed if and only if there exists a fuzzy set A of X such that γ-cl(int(A))≤B≤A.

Proof: (1) Let B be a fuzzy subset of X. If a fuzzy set A of X such that A≤B≤ γ-int(cl(A)) exists, then B≤ γ-int(cl(A)) ≤ γ-int(cl(B)). Thus B is a fuzzy γ-pre open set. Conversely if B is any fuzzy γ-pre open set, then the result follows for A= B. Hence proved (1).
(2) Let B be a fuzzy subset of X. If a fuzzy set A of X such that γ-cl(int(A))≤B≤A exists, then A≥B≥ γ-cl(int(A))≥ γ-cl(int(B)). Thus B is a fuzzy γ-pre closed set. Conversely if B is any fuzzy γ-pre closed set, then the result follows for A= B. Hence proved (2).

Theorem 6.6: Let A be a fuzzy set of a fuzzy topological space X.

(1) If A is fuzzy strongly pre open set, then γ-pcl(A)=A.

(2) If A is fuzzy strongly pre closed set, then γ-pint(A) =A.

Proof: Let A be fuzzy strongly pre open. Then by Definition 2.1, A≤ int(pcl(A)). That is cl(A)≤cl(int(pcl(A))) ≤ pcl(A) ≤ γ-pcl(A) and since γ-pcl(A)≤ A. Thus γ-pcl(A) = A. This proves (1). Let A be fuzzy strongly pre closed set. Then A^c is fuzzy strongly pre open. By (1), γ-pcl(A^c) = cl(A^c). By 6.2(i), $[\gamma\text{-pint}(A)]^c = [\text{int}(A)]^c$. Thus γ-pint(A) =A. Hence (2).

Proposition 6.7: Let (X, τ) be a fuzzy topological space. Then for a fuzzy subset A and B of a fuzzy topological space X, we have

(i) γ-pcl ($A\vee B$) = γ-pcl (A) $\vee$ γ-pcl (B) and

(ii) γ-pcl ($A\wedge B$) ≤ γ-pcl (A) $\wedge$ γ-pcl (B).

Proof: Since γ-pcl ($A\vee B$) = $\gamma\text{-pcl}[(A \vee B)^c]^c$, by using Proposition 6.2(i), we have
$\gamma\text{-pcl }(A\vee B) = [\gamma\text{-pint }(A\vee B)^c]^c = [\gamma\text{-pint }(A^c\wedge B^c)]^c$. Again using Proposition 5.3(i), we have $\gamma\text{-pcl }(A\vee B) = [\gamma\text{-pint }(A^c)\wedge \gamma\text{-pint }(B^c)]^c = [\gamma\text{-pint }(A^c)]^c \vee [\gamma\text{-pint }(B^c))]^c$. By using Proposition 6.2(i), we have $\gamma\text{-pcl }(A\vee B) = \gamma\text{-pcl }(A^c)^c \vee \gamma\text{-pcl }(B^c)^c$ = γ-pcl (A) $\vee$ γ-pcl (B). Thus proved (i). Since $A \wedge B \leq A$ and $A \wedge B \leq B$, by using Proposition 6.3(iv), γ-pcl ($A\wedge B$) ≤ γ-pcl(A) and γ-pcl($A\wedge B$) ≤ γ-pcl (B). This implies that γ-pcl ($A\wedge B$) ≤ γ-pcl (A) $\wedge$ γ-pcl (B). This proves(ii).

The following example shows that γ-pcl ($A\wedge B$) need not be equal to γ-pcl(A) $\wedge$ γ-pcl(B).

Example 6.8: Let X = {a, b, c} and τ ={0, 1, {$a_{.4}$, $b_{.3}$, $c_{.2}$}, {$a_{.6}$, $b_{.2}$, $c_{.1}$}, {$a_{.4}$, $b_{.2}$, $c_{.1}$}, {$a_{.6}$, $b_{.3}$, $c_{.2}$}}. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is τ^c = {0, 1, {$a_{.6}$, $b_{.7}$, $c_{.8}$}, {$a_{.4}$, $b_{.8}$, $c_{.9}$}, {$a_{.6}$, $b_{.8}$, $c_{.9}$}, {$a_{.4}$, $b_{.7}$, $c_{.8}$}}. Consider A = {$a_{.6}$, $b_{.3}$, $c_{.4}$} and B = {$a_{.5}$, $b_{.3}$, $c_{.5}$}. Then γ-pcl(A) = {$a_{.6}$, $b_{.4}$, $c_{.5}$} and γ-pcl (B) = {$a_{.5}$, $b_{.5}$, $c_{.6}$}. Also γ-pcl (A) $\wedge$ γ-pcl (B) = {$a_{.5}$, $b_{.4}$, $c_{.5}$}. Now $A \wedge B$ = {$a_{.5}$, $b_{.3}$, $c_{.4}$} and γ-pcl ($A\wedge B$) = {$a_{.5}$, $b_{.4}$, $c_{.4}$}. Thus γ-scl (A) $\wedge$ γ-scl (B) ≠ γ-scl ($A\wedge B$).

Theorem 6.9: Let (X, τ) be a fuzzy topological space. Then for any family {A_α} of fuzzy subsets of a fuzzy topological space we have,

(i) $\vee[\gamma\text{-pcl}(A_\alpha)] \leq \gamma\text{-pcl}(\vee A_\alpha)$.

(ii) $\gamma\text{-pcl}(\wedge A_\alpha) \leq \wedge[\gamma\text{-pcl}(A_\alpha)]$.

Proof: For every β, $A_\beta \leq \vee A_\alpha \leq \gamma$-pcl($\vee A_\alpha$). By using Proposition 6.3(iv), γ-pcl(A_β) $\leq \gamma$-pcl($\vee A_\alpha$) for every β. This implies that $\vee$(γ-pcl(A_β)) $\leq \gamma$-pcl($\vee A_\alpha$). This proves (i).
Now $\wedge A_\alpha \leq A_\beta$ for every β. Again using Proposition 6.3, we get γ-pcl($\wedge A_\alpha$) $\leq \gamma$-pcl(A_β). This implies that γ-pcl($\wedge A_\alpha$) $\leq \wedge$[γ-pcl(A_β)].

Theorem 6.10: Let (X, τ) be a fuzzy topological space. Then for a fuzzy subset A and B of X we have,

(i) γ-pcl (A) $\geq$ A $\vee$ cl (int (A)).
(ii) γ-pint (A) $\leq$ A$\wedge$int (cl (A)).
(iii) int (γ-pcl (A)) $\leq$ int (cl (A)).
(iv) int (γ-pcl (A)) $\geq$ int (cl(int (A))).

Proof: (i) Since pcl(A) is a fuzzy pre closed set, we have cl(int(A))$\leq$cl(int(pcl(A)))$\leq$pcl(A)$\leq \gamma$-pcl (A) since A$\leq \gamma$-pcl (A). Thus A $\vee$ cl (int (A)) $\leq \gamma$-pcl (A). This proves (i).
(ii) Since γ-pint (A) is fuzzy γ –pre open set, γ-pint (A) $\leq$ A. Then cl(γ-pint (A)) $\leq$ cl(A).
This implies that γ-int(cl(γ-pint (A))) $\leq \gamma$-int (cl(A)). It follows that
γ-pint(A) $\leq \gamma$-int(cl(A))) $\leq$ int(cl(A)). It shows that γ-pint (A) $\leq$ A$\wedge$int (cl (A)).
(i) Since γ-pcl (A)$\leq$A, int(γ-pcl (A))$\leq$int(cl(A)). This proves (iii).
(ii) By (i), int(γ-pcl (A)) $\geq$ int(A $\vee$ cl (int (A)))$\geq$int(cl(int(A))).

The family of all fuzzy pre open (fuzzy pre closed, fuzzy strongly pre open, fuzzy strongly pre closed, fuzzy γ-pre open, fuzzy γ-pre closed, fuzzy γ-open, fuzzy γ-closed) sets of an fuzzy topological space(X, τ) will be denoted by Fso(τ) (Fscl(τ), Fsso(τ), Fsscl(τ), Fγso(τ), Fγscl(τ), Fγo(τ), Fγcl(τ)).

Proposition 6.11: Let (X, τ) be a fuzzy topological space. Then

1) Fγo(τ) $\wedge$ Fpo(τ) $\leq$ Fγpo(τ).
2) Fγcl(τ) $\wedge$ Fpcl(τ) $\leq$ Fγpcl(τ).
3) Fpo(τ) $\wedge$ Fspo(τ) $\leq$ Fγpo(τ).
4) Fpcl(τ) $\wedge$ Fspcl(τ)$\leq$ Fγpcl(τ).

Proof: Let A be a fuzzy subset of Fγo(τ) $\wedge$ Fpo(τ). Then A $\in$ F γ o(τ) and A $\in$ Fpo(τ). By Definition 2.1, A $\leq$ cl(int(A)) int(cl(A)) $\leq$ int(cl(A))$\leq \gamma$-int(cl(A)).Since A $\in$ Fpo(τ), we have A $\leq$ int(cl(A)) $\leq\gamma$-int(cl(A)). Therefore A $\in$ F γpo(τ). This proves (1).

Let A be a fuzzy subset of Fγcl(τ) $\wedge$ Fpcl(τ). Then A $\in$ Fγcl(τ) and A $\in$ Fpcl(τ). By Definition 2.1, A $\geq$ cl(int(A))$\wedge$ int(cl(A)) $\geq$cl(int(A))$\geq \gamma$-cl(int(A)). Since A $\in$ Fpcl(τ), we have A$\geq$cl(int(A))$\geq\gamma$-cl(int(A)). Therefore A $\in$ F γpcl(τ). This proves (2).

Let A$\in$Fpo(τ) $\wedge$ Fspo(τ). Then A $\in$ Fpo(τ) and A $\in$ Fspo(τ). By the Definition of 2.1, A$\leq$ int(cl(A)) $\leq \gamma$-int (cl(A)). Since A $\in$ Fspo(τ), A$\leq$ int(pcl(A)) $\leq$ int(cl(A))$\leq \gamma$-int(cl(A)). Therefore A is fuzzy γ-pre open. This proves (3).

Let A$\in$Fpcl(τ) $\wedge$ Fspcl(τ). Then A $\in$ Fpcl(τ) and A $\in$ Fspcl(τ). By the Definition of 2.1, A$\geq$ cl(intA)) $\leq \gamma$-cl(int(A)). Since A $\in$ Fspcl(τ), A$\geq$cl(pint(A)) $\geq$ cl(int(A))$\leq \gamma$-cl(int(A)). Hence proved (4).

Definition 6.12: An fuzzy topological space (X, τ) is fuzzy γ-PO-extremely disconnected if and only if γ-pcl(A) is a fuzzy γ-pre open set, for each fuzzy γ-pre open set A of (X, τ).

Theorem 6.13: Let (X, τ) be an fuzzy topological space. Then the following statements are equivalent:

(i) X is γ-PO-extremely disconnected.
(ii) γ-pint(A) is a fuzzy γ-pre closed set, for each fuzzy γ-pre closed set A of X.
(iii) γ-pcl(γ-pcl (A))^c = (γ-pcl (A))^c, for each fuzzy γ-pre open set A of X.
(iv) B = (γ-pcl (A))^c implies γ-pcl(B)=(γ-pcl (A))^c for each pair of fuzzy γ- pre open sets A, B of X.

Proof: (i) $\Rightarrow$ (ii) Let A be a fuzzy γ-pre closed set of X. Then A^c is a fuzzy γ-pre open set.
According to the assumption, γ-pcl (A^c) is fuzzy γ-pre open set. So γ-pint (A) is a fuzzy γ-pre

closed set of X. (ii) $\Rightarrow$ (iii) Suppose that A is a fuzzy γ-pre open set of X. Then γ-pcl(γ-pcl $(A))^c$ = γ-pcl (γ-pint($A)^c$). According to the assumption, γ-pint (A^c) is a fuzzy γ-pre closed set. So γ-pcl (γ-pint (A^c)) = γ-pint (A^c) = (γ-pcl $(A))^c$. (iii)$\Rightarrow$ (iv) Let A and B be a fuzzy γ-pre open set of X such that B =(γ-pcl $(A))^c$. From the assumption we have, γ-pcl B = γ-pcl(γ-pcl $(A))^c$ =(γ-pcl $(A))^c$. (iv) $\Rightarrow$ (i) Let A be a fuzzy γ-pre open set of X. We put B = (γ-pcl $(A))^c$ From the assumption, we obtain that γ-pcl (B) = (γ-pcl $(A))^c$, so (γ-pcl $(B))^c$ = γ-pcl(A). Hence γ-pint (B^c) = γ-pcl(A). Thus γ-pcl(A) is fuzzy γ-pre open set of X.

Definition 6.14: A fuzzy set A of fuzzy topological space (X, τ) is said to be

(a) fuzzy γ-t-set if int (A)=int(γ-cl(A)).

(b) fuzzy γ-m-set if A = U∧V, where U is fuzzy open and V is fuzzy γ-t-set.

Theorem 6.15: Let(X, τ) be a fuzzy topological space. Then a fuzzy open subset A is fuzzy γ-pre open and fuzzy γ-m-set.

Proof: Let A be a fuzzy open set. Then int(A) = A. In addition A ≤ cl(A), that implies A= int(A)≤ int(cl(A))≤ γ-int(cl(A)). Hence A is fuzzy γ-pre open. More over A = A∧1_x. By Definition 6.14, A is fuzzy γ-m-set.

Theorem 6.16: Let (X, τ) be an fuzzy topological space. If A is fuzzy γ-closed, then it is fuzzy γ-t-set.

Proof: Let A be fuzzy γ-closed. Then by Proposition 2.10,A=γ-cl(A) and int(A)=int(γ-cl(A)). Therefore A is fuzzy γ-t-set.

Theorem 6.17: Let (X, τ) be an fuzzy topological space. Then the intersection of any two fuzzy γ-t-set is fuzzy γ-t-set.

Proof: Let A and B be fuzzy γ-t-set. Then by Definition 6.14, int(A)=int(γ-cl(A)) and int(B)=int(γ-cl(B)).Therefore int(A)∧int(B)=int(γ-cl(A)) ∧ int(γ-cl(B)) = int(γ-cl(A) ∧γ-cl(B)). By Remark 2.6, int(A∧B) = int(γ-cl(A ∧ B)).

The following example shows that union of two fuzzy γ-t-set need not be fuzzy γ-t-set.

Example 6.15: Let X = {a, b} and τ = {0, 1, {$a_{.1}$, $b_{.3}$}, {$a_{.9}$, $b_{.7}$}}. Then (X, τ) is a fuzzy topological space. The family of all fuzzy closed sets of τ is τ^c = {0, 1, {$a_{.9}$, $b_{.7}$}, {$a_{.1}$, $b_{.3}$}}. Consider A = {$a_{.4}$, $b_{.7}$} and B = {$a_{.8}$, $b_{.6}$}. Then int(A)= {$a_{.1}$, $b_{.3}$} and int(B)= {$a_{.1}$, $b_{.3}$}. It follows that int(γ-cl(A))= {$a_{.1}$, $b_{.3}$}and int(γ-cl(B))= {$a_{.1}$, $b_{.3}$}. Therefore by Definition 6.10, A and B are fuzzy γ-t-set. Now A∨B={$a_{.8}$, $b_{.7}$} and int (A∨B)= {$a_{.1}$, $b_{.3}$} but int(γ-cl(A∨B))= {$a_{.9}$, $b_{.7}$}. It shows that A∨B is not an fuzzy γ- t-set.

Proposition 6.16: Let (X, τ) be a fuzzy topological space. Let A be a fuzzy subset of X. Then

(i) γ-cl(A) = int(cl(A)).

(ii) int(γ-cl(A))=int(cl(A)).

Proof: Since A is fuzzy γ-closed ⇔ γ-cl(A)=A. Therefore γ-cl(A)≥int(cl(A))∧cl(int(A)) and γ-cl(A)≥A. Then γ-cl(A)≥A∨(int(cl(A))∧cl(int(A))). If A is fuzzy γ-closed, then γ-cl(A)≤A. Since A is fuzzy open,int(A)=A. Since A≤cl(A), we have A≤int(cl(A)). Since A≤cl(A), we have A ≤ cl(int(A)). Thus γ-cl(A)≤A≤int(cl(A))∧cl(int(A)). Thereforeγ-cl(A)≤A∨int(cl(A))∧cl(int(A)).

This implies that γ-cl(A)=A∨(int(cl(A))∧cl(int(A)))=A∨int(cl(A))=int(cl(A)). This proves (i). By (i), γ-cl(A) = int(cl(A)). It follows that int(γ-cl(A))=int(cl(A)).

Refrences

[1] M.Athar and B.Ahmad, Fuzzy Boundary and Fuzzy Semiboundary, *Advances in Fuzzy systems*, vol. 2008, 586893, 9 pages.

[2] K.K.Azad, On fuzzy semi-continuity, fuzzy almost continuity and fuzzy weakly continuity, *J.Math.Anal.Appl*.

82(1)(1981), 14-32.

[3] Biljana Krsteska , Fuzzy Strongly preopen sets and fuzzy strongly precontinuity, *Mat. Vesnik* 50(1998),111-123.

[4] C.L.Chang, Fuzzy Topological Spaces, J.Math. Anal, Appl.24(1968), 182-190.

[5] Luay A.Al.Swidi, Amed S.A.Oon, Fuzzy γ-open sets and fuzzy γ-closed sets, *Americal Journal of scientific research*, 27(2011), 62-67.

[6] T.Noiri and O.R.Sayed, Fuzzy γ-open sets and fuzzy γ-continuity in fuzzitying topology, *Sci. Math. Jpn*. 55(2002).

[7] A.K. Katsaras and D.B. Liu, Fuzzy vector spaces and fuzzy topological vector spaces , *J.Math.Anal.Appl*. vol. 58,no 1, pp. 135-146, 1977.

[8] M.K.Singal and N.Prakash, Fuzzy preopen sets and fuzzy preseparation axioms, Fuzzy Sets and Systems 44(1991) 273-281.

[9] R.Usha Parameswari and K.Bageerathi, On Fuzzy γ - Semi Open Sets and Fuzzy γ - Semi Closed Sets in Fuzzy Topological Spaces, *International Organisation of Scientific and Research*, 7(1),(2013), 63 - 70.

[10] L.A.Zadch, *Fuzzy Sets*, Information and control, vol.8, Pp 338-353, 1965.