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RESEARCH ARTICLE

A New Generalized Yang-Fourier Transforms to Heat-Conduction in a Semi-Infinite Fractal Bar

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Abstract:

The purpose of present paper to solve 1-D fractal heat-conduction problem in a fractal semi-infinite bar has been developed by local fractional calculus employing the analytical Manoj Generalized Yang-Fourier transforms method.

Key Words: fractal bar, heat-conduction equation, A New Generalized Yang-Fourier transforms, Yang-Fourier transforms, local fractional calculus.

1. Introduction

A New Generalized Yang-Fourier transforms which is obtained by authors by generalization of Yang-Fourier transforms is a technique of fractional calculus for solving mathematical, physical and engineering problems. The fractional calculus is continuously growing in last five decades [1-7]. Most of the fractional ordinary differential equations have exact analytic solutions, while others required either analytical approximations or numerical techniques to be applied, among them: fractional Fourier and Laplace transforms [8,41], heat-balance integral method [9-11], variation iteration method (VIM) [12-14], decomposition method [15,41], homotopy perturbation method [16,41] etc.

The problems in fractal media can be successfully solved by local fractional calculus theory with problems for non-differential functions [25-32]. Local fractional differential equations have been applied to model complex systems of fractal physical phenomena [30-41] local fractional Fourier series method [38], Yang-Fourier transform [39, 40,41]

2. Generalized Yang-Fourier transform and its properties:

Let us Consider $f(x)$ is local fractional continuous in $(-\infty,\infty)$ we denote as $f(x) \in Ca,\beta(-\infty,\infty)$ [32, 33, 35].

Let $f(x) \in Ca,\beta(-\infty,\infty)$ A New Generalized Yang-Fourier transform developed by authors is written in the form [30, 31, 39, 40, 41]:

$$F_{\alpha,\beta}\{f(x)\} = f_\omega^{F,\alpha,\beta}(\omega) = {}_p^0M_q^{\alpha,\beta}\left(a_1 \dots a_p; b_1 \dots b_q; z\right)$$

$$= \sum_{n=o}^{\infty} \frac{(a_1)_n \dots (a_p)_n}{(b_1)_n \dots (b_q)_n} \frac{1}{\Gamma(1+\alpha+\beta)} \int_{-\infty}^{\infty} {}_p^0M_q^{\alpha,\beta}\left(-i^{\alpha+\beta}\omega^{\alpha+\beta}x^{\alpha+\beta}\right) f(x)(dx)^{\alpha+\beta} \qquad (1)$$

When we put β equal to zero, and if there is no upper and lower parameter in (1) it converts in to the Yang-Fourier transform [41].

Then, the local fractional integration is given by [30-32, 35-37, 41]:

$$\sum_{n=o}^{\infty} \frac{(a_1)_n \dots (a_p)_n}{(b_1)_n \dots (b_q)_n} \frac{1}{\Gamma(1+\alpha+\beta)} \int_a^b f(t)\,(dx)^{\alpha+\beta} = \frac{1}{\Gamma(1+\alpha+\beta)} \lim_{\Delta t\to 0} \sum_{j=0}^{j=N-1} f(t_j)\,(\Delta t_j)^{\alpha+\beta} \qquad (2)$$

where $\Delta t_j = t_{j+1} - t_j$, $\Delta t = max\{\Delta t_1, \Delta t_2, \Delta t_j, \dots\}$ and $\{t_j, t_{j+1}\}$, $j = 0, \dots, N-1$, t_0 = a, $t_N = b$, is a partition of the interval $[a, b]$.

If $F_{\alpha,\beta}\{f(x)\} = f_\omega^{F,\alpha,\beta}(\omega)$, then its inversion formula takes the form [30, 31, 39, 40,41]

$$f(x) = F_{\alpha,\beta}^{-1}\left[f_\omega^{F,\alpha,\beta}(\omega)\right]$$

$$= \sum_{n=o}^{\infty} \frac{(a_1)_n \ \dots \ (a_p)_n}{(b_1)_n \dots (b_q)_n} \frac{1}{(2\pi)^{\alpha+\beta}} \int_{-\infty}^{\infty} {}_pM_q^{\alpha,\beta}\left(-i^{\alpha+\beta}\,\omega^{\alpha+\beta}\,x^{\alpha+\beta}\right) f_\omega^{F,\alpha,\beta}(\omega)(d\omega)^{\alpha+\beta} \qquad (3)$$

When we put β equal to zero, and if there is no upper and lower parameter it converts in to the Yang Inverse Fourier transform [41].

Some properties are shown as it follows [30, 31]:
Let $F_{\alpha,\beta}\{f(x)\} = f_\omega^{F,\alpha,\beta}(\omega)$, and $F_{\alpha,\beta}\{g(x)\} = f_\omega^{F,\alpha,\beta}(\omega)$, and let be two constants, $if\ (\delta)_0$. Then we have:

$$F_{\alpha,\beta}\{cf(x) + dg(x)\} = cF_{\alpha,\beta}\{f(x)\} + dF_{\alpha,\beta}\{g(x)\} \qquad (4)$$

If $\lim_{|x|\to\infty} f(x) \ = \ 0$, then we have:

$$F_{\alpha,\beta}\{f^{\alpha,\beta}(x)\} = i^{\alpha+\beta}\,\omega^{\alpha+\beta} F_{\alpha,\beta}\{f(x)\} \qquad (5)$$

In eq. (5) the local fractional derivative is defined as:

$$f^{\alpha,\beta}(x_0) = \left.\frac{d^{\alpha+\beta} f(x)}{dx^{\alpha+\beta}}\right|_{x=x_0} = \lim_{x\to x_0} \frac{\Delta^{\alpha+\beta}[f(x) - f(x_0)]}{(x - x_0)^{\alpha+\beta}} \qquad (6)$$

Where $\Delta^{\alpha+\beta}[f(x) - f(x_0)] \cong \Gamma(1 + \alpha + \beta)\Delta[f(x) - f(x_0)]$,
As a direct result, repeating this process, when:

$$f(0) = f^{\alpha,\beta}(0) = \dots = f^{(k-1)\alpha,(k-1)\beta}(0) = 0 \qquad (7)$$

$$F_{\alpha,\beta}\{f^{k\alpha,k\beta}(x)\} = i^{\alpha+\beta}\,\omega^{\alpha+\beta} F_{\alpha,\beta}\{f(x)\} \qquad (8)$$

3. Heat conduction in a fractal semi-infinite bar:

If a fractal body is subjected to a boundary perturbation, then the heat diffuses in depth modeled by a constitutive relation where the rate of fractal heat flux $\overline{q}(x, y, z, t)$ is proportional to the local fractional gradient of the temperature [32,41], namely:

$$\overline{q}(x, y, z, t) = -K^{2\alpha+2\beta}\nabla^{\alpha+\beta} T(x, y, z, t) \qquad (9)$$

Here the pre-factor $K^{2\alpha+2\beta}$ is the thermal conductivity of the fractal material. Therefore, the fractal heat conduction equation without heat generation was suggested in [32] as:

$$K^{2\alpha+2\beta} \frac{d^{2(\alpha+\beta)} T(x, y, z, t)}{dx^{2(\alpha+\beta)}} - \rho_{\alpha+\beta}\, c_{\alpha+\beta} \frac{d^{2(\alpha+\beta)} T(x, y, z, t)}{dx^{2(\alpha+\beta)}} = 0 \qquad (10)$$

Where $\rho_{\alpha+\beta}$ and $c_{\alpha+\beta}$ are the density and the specific heat of material, respectively.
The fractal heat-conduction equation with a volumetric heat generation $g(x, y, z, t)$ can be described as [32,41]:

$$K^{2\alpha+2\beta}\nabla^{2\alpha+2\beta} T(x, y, z, t) + g(x, y, z, t)\rho_{\alpha+\beta}\, c_{\alpha+\beta} \frac{\partial^{(\alpha+\beta)} T(x, y, z, t)}{\partial t^{(\alpha+\beta)}} \qquad (11)$$

The 1-D fractal heat-conduction equation [32,41] reads as:

$$K^{2\alpha+2\beta} \frac{\partial^{2(\alpha+\beta)} T(x, t)}{\partial x^{2(\alpha+\beta)}} - \rho_{\alpha+\beta}\, c_{\alpha+\beta} \frac{\partial^{(\alpha+\beta)} T(x, t)}{\partial t^{(\alpha+\beta)}} = 0, \qquad 0 < x < \infty, t > 0 \qquad (12a)$$

with initial and boundary conditions are:

$$\frac{\partial^{(\alpha+\beta)} T(0, t)}{\partial t^{(\alpha+\beta)}} = {}_pM_q^{\alpha,\beta}\, t^{\alpha+\beta}, T(0, t) = 0 \qquad (12b)$$

The dimensionless forms of (12a, b) are [35, 41]:

$$\frac{\partial^{2(\alpha+\beta)} T(x, t)}{\partial x^{2(\alpha+\beta)}} = \frac{\partial^{(\alpha+\beta)} T(x, t)}{\partial x^{(\alpha+\beta)}} = 0 \qquad (13a)$$

$$\frac{\partial^{(\alpha+\beta)} T(0, t)}{\partial x^{(\alpha+\beta)}} = {}_pM_q^{\alpha,\beta}\, t^{\alpha+\beta}, T(0, t) = 0 \qquad (13b)$$

Based on eq. (12a), the local fractional model for 1-D fractal heat-conduction in a fractal semi-infinite bar with a source term $g(x, t)$ is:

$$K^{2\alpha+2\beta} \frac{\partial^{2(\alpha+\beta)} T(x, t)}{\partial x^{2(\alpha+\beta)}} - \rho_{\alpha+\beta}\, c_{\alpha+\beta} \frac{\partial^{(\alpha+\beta)} T(x, t)}{\partial t^{(\alpha+\beta)}} = g(x, t), \qquad -\infty < x < \infty, t > 0 \qquad (14a)$$

With

$$T(x, 0) = f(x), -\infty < x < \infty, \qquad (14b)$$

The dimensionless form of the model (14a, b) is:

$$\frac{\partial^{2(\alpha+\beta)}T(x,t)}{\partial x^{2(\alpha+\beta)}}=\frac{\partial^{(\alpha+\beta)}T(x,t)}{\partial t^{(\alpha+\beta)}}=0, \qquad -\infty<x<\infty, t>0 \tag{15a}$$

$$T(x,0)=f(x), -\infty<x<\infty, \tag{15b}$$

4. Solutions by the Generalized Yang-Fourier transform method:

Let us consider that $F_{\alpha,\beta}\{T(x,t)\}=T_{\omega}^{F,\alpha,\beta}(\omega,t)$ is the Generalized Yang-Fourier transform of $T(x, t)$, regarded as a non-differentiable function of x. Applying the Yang-Fourier transform to the first term of eq. (15a), we obtain:

$$F_{\alpha,\beta}\left\{\frac{\partial^{2(\alpha+\beta)}T(x,t)}{\partial x^{2(\alpha+\beta)}}\right\}=\left(i^{2(\alpha+\beta)}\omega^{2(\alpha+\beta)}\right)T_{\omega}^{F,\alpha,\beta}(\omega,t)=\omega^{2(\alpha+\beta)}T_{\omega}^{F,\alpha,\beta}(\omega,t) \tag{16a}$$

On the other hand, by changing the order of the local fractional differentiation and integration in the second term of eq.(15a), we get:

$$F_{\alpha,\beta}\left\{\frac{\partial^{2(\alpha+\beta)}}{\partial t^{2(\alpha+\beta)}}T(x,t)\right\}=\frac{\partial^{(\alpha+\beta)}}{\partial t^{(\alpha+\beta)}}T_{\omega}^{F,\alpha,\beta}(\omega,t) \tag{16b}$$

For the initial value condition, the Yang-Fourier transform provides:

$$F_{\alpha,\beta}\{T(x,0)\}=T_{\omega}^{F,\alpha,\beta}(\omega,0)=F_{\alpha,\beta}\{f(x)\}=f_{\omega}^{F,\alpha,\beta}(\omega) \tag{16c}$$

Thus we get from eqn. (16a, b, c):

$$\frac{\partial^{(\alpha+\beta)}}{\partial t^{(\alpha+\beta)}}T_{\omega}^{F,\alpha,\beta}(\omega,t)+\omega^{2(\alpha+\beta)}T_{\omega}^{F,\alpha,\beta}(\omega,t)=0, T_{\omega}^{F,\alpha,\beta}(\omega,0)=f_{\omega}^{F,\alpha,\beta}(\omega) \tag{17}$$

This is an initial value problem of a local fractional differential equation with t as independent variable and w as a parameter.

$$T(\omega,t)=f_{\omega}^{F,\alpha,\beta}(\omega)\,{}_pM_q^{\alpha,\beta}\left(-\omega^{2(\alpha+\beta)}t^{\alpha+\beta}\right) \tag{18a}$$

Consequently, using inversion formula, eqn. (3), we obtain:

$$T(x,t)=\sum_{n=o}^{\infty}\frac{(a_1)_n\ \dots\ (a_p)_n}{(b_1)_n\dots(b_q)_n}\frac{1}{(2\pi)^{\alpha+\beta}}\int_{-\infty}^{\infty}{}_pM_q^{\alpha,\beta}\left(i^{\alpha+\beta}\omega^{\alpha+\beta}x^{\alpha+\beta}\right)f_{\omega}^{F,\alpha,\beta}(\omega)\,{}_pM_q^{\alpha,\beta}\left(-\omega^{2(\alpha+\beta)}t^{\alpha+\beta}\right)(d\omega)^{\alpha+\beta} \tag{18b}$$

$$M_{\omega}^{F,\alpha,\beta}(\omega)=\frac{1}{(2\pi)^{\alpha+\beta}}\,{}_pM_q^{\alpha,\beta}\left(-\omega^{2(\alpha+\beta)}t^{\alpha+\beta}\right) \tag{18c}$$

From [30, 32] we obtained,

$$F_{\alpha+\beta}\left\{{}_pM_q^{\alpha,\beta}\left(-\frac{\omega^{2(\alpha+\beta)}}{C^{2(\alpha+\beta)}}\right)\right\}=\frac{C^{(\alpha+\beta)}\pi^{\frac{\alpha+\beta}{2}}}{\Gamma(1+\alpha+\beta)}\,{}_pM_q^{\alpha,\beta}\left(-\frac{C^{2(\alpha+\beta)}\omega^{2(\alpha+\beta)}}{4^{(\alpha+\beta)}}\right) \tag{19a}$$

Let $C^{2(\alpha+\beta)}/4^{\alpha+\beta}=t^{\alpha+\beta}$. Then we get:

$$F_{\alpha+\beta}\left\{{}_pM_q^{\alpha,\beta}\left(-\frac{\omega^{2(\alpha+\beta)}}{4^{\alpha+\beta}\ t^{\alpha+\beta}}\right)\right\}=\frac{4^{\alpha+\beta}t^{\frac{\alpha+\beta}{2}}\pi^{\frac{\alpha+\beta}{2}}}{\Gamma(1+\alpha+\beta)}\,{}_pM_q^{\alpha,\beta}\left(-\omega^{2(\alpha+\beta)}t^{\alpha+\beta}\right)$$

$$=\sum_{n=o}^{\infty}\frac{(a_1)_n\ \dots\ (a_p)_n}{(b_1)_n\dots(b_q)_n}\frac{4^{\alpha+\beta}t^{\frac{\alpha+\beta}{2}}\pi^{\frac{\alpha+\beta}{2}}}{\Gamma(1+\alpha+\beta)}(2\pi)^{\alpha+\beta}M_{\omega}^{F,\alpha,\beta}(\omega) \tag{19b}$$

Thus, $M_{\omega}^{F,\alpha,\beta}(\omega)$ have the inverse:

$$\sum_{n=o}^{\infty}\frac{(a_1)_n\ \dots\ (a_p)_n}{(b_1)_n\dots(b_q)_n}\frac{1}{(2\pi)^{\alpha+\beta}}\int_{-\infty}^{\infty}{}_pM_q^{\alpha,\beta}\left(i^{\alpha+\beta}\omega^{\alpha+\beta}x^{\alpha+\beta}\right)M_{\omega}^{F,\alpha,\beta}(\omega)(d\omega)^{\alpha+\beta}$$

$$=\frac{\Gamma(1+\alpha+\beta)}{4^{\alpha+\beta}t^{\frac{\alpha+\beta}{2}}\pi^{\frac{\alpha+\beta}{2}}}\sum_{n=o}^{\infty}\frac{(a_1)_n\ \dots\ (a_p)_n}{(b_1)_n\dots(b_q)_n}\frac{1}{(2\pi)^{\alpha+\beta}}\,{}_pM_q^{\alpha,\beta}(\alpha+\beta)\ \left(-\frac{\omega^{2(\alpha+\beta)}}{4^{\alpha+\beta}\ t^{\alpha+\beta}}\right) \tag{19c}$$

Hence, we get:

$$T(x,t) = (Mf)(x)$$

$$= \sum_{n=o}^{\infty} \frac{(a_1)_n \ \dots \ (a_p)_n}{(b_1)_n \dots (b_q)_n} \frac{\Gamma(1+\alpha+\beta)}{4^{\alpha+\beta} t^{\frac{\alpha+\beta}{2}} \pi^{\frac{\alpha+\beta}{2}}} \int_{-\infty}^{\infty} f(\xi)_p M_q^{\alpha,\beta} \left(-\frac{(x-\xi)^{2(\alpha+\beta)}}{4^{\alpha+\beta} t^{\alpha+\beta}}\right) (d\xi)^{\alpha+\beta} \qquad (20)$$

Special case

If we take $\beta = 0$ and if there is no upper and lower parameter then the results of a New generalized Yang Fourier Transforms convert in Yang Fourier Transforms results [41]

Conclusions

The communication, presented an analytical solution of 1-D heat conduction in fractal semi-infinite bar by the A New Generalized Yang-Fourier transform of non-differentiable functions.

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